

Alternative interpretation of $E0$ strengths in transitional regions

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Abstract. A strong rise of $E0$ transition strengths between the first excited 0^+ state and the ground state is predicted in shape transitional regions within the Interacting Boson Model (IBM). This rise matches well existing data and is not connected to a large mixing amplitude between both states. Moreover, a coherence of amplitudes in the wave functions causes the strong transition, without a requirement of explicit mixing of normal and intruder configurations.

PACS. 21.60.Ev Collective models – 21.10.Ky Electromagnetic moments – 21.60.Fw Models based on group theory

1 Introduction

The investigation of shape/phase transitions in nuclei has so far been focused on $E2$ properties. There has only been little study of $E0$ matrix elements, despite that the $E0$ operator $\rho(E0)$ is directly connected to changes in nuclear shapes and radii. We will use the interacting boson model (IBM-1) [1] for a survey of $E0$ properties among 0^+ states. Commonly, large $E0$ transition strengths between the 0_1^+ and 0_2^+ states are modeled by an explicit mixing of coexisting spherical and deformed intruder configurations [2]. However, our calculations show that large $E0$ strengths do not require such explicit mixing, they are rather inherent to the model [3].

2 IBM-1 calculations

We used the simple Ising-type two-parameter Hamiltonian [4]

$$H = a \left[(1 - \zeta)n_d - \frac{\zeta}{4N} Q \cdot Q \right], \quad (1)$$

with the quadrupole operator $Q = s^\dagger \tilde{d} + d^\dagger s + \chi(d^\dagger \tilde{d})^{(2)}$ and boson number N . For $\zeta = 0$ one obtains the $U(5)$ limit while $\zeta = 1$ and $\chi = -\sqrt{7}/2$ gives $SU(3)$, and $\zeta = 1$ and $\chi = 0$ gives $O(6)$. Therefore, using the $E0$ operator

$$\rho(E0) = \alpha N + \beta' (d^\dagger \tilde{d})^{(0)}, \quad (2)$$

$E0$ strengths between 0^+ states can be calculated over a wide range of symmetries, including those parameter regions (around $\zeta = 0.5$) that are known to show phase transitional behavior. The top part of fig. 1 shows $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$ for $N = 16$ bosons, which shows a sharp rise just in the parameter region of the vibrator-rotor shape/phase transition. $E0$ strength even remains large in the rotational limit. The drop at $O(6)$ is due to an exchange of the 0_2^+ and 0_3^+ states —if both matrix elements are added (fig. 1 bottom part for $N = 10$ bosons) it is seen that the $E0$ strength remains large on the deformed side, as well for axially symmetric deformation as for γ -soft triaxial deformation.

From a more detailed analysis it is seen that while large $E0$ strengths in the IBM-1 are connected to components with large n_d values in the wave functions, the appearance of such large n_d values alone is not sufficient. There are subtle cancellation effects of positive and negative parts in the matrix elements, ending up in a large $E0$ strength to the ground state only for one excited 0^+ state.

3 Comparison with data

The robust prediction of large $E0$ strengths in the few-parameter IBM-1 needs experimental testing. $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$ values are known [2] in the $A = 100$ and 150 transition regions. Figure 2 compares these data with a schematic IBM calculation, where with fixed parameters $N = 10$, $\chi = -\sqrt{7}/2$, and $\beta' = 6 \times 10^{-3/2}/eR_0^2$ (note the incorrect equation in ref. [3]). Only the parameter ζ was allowed to vary and was fitted to the $R_{4/2} = E(4_1^+)/E(2_1^+)$

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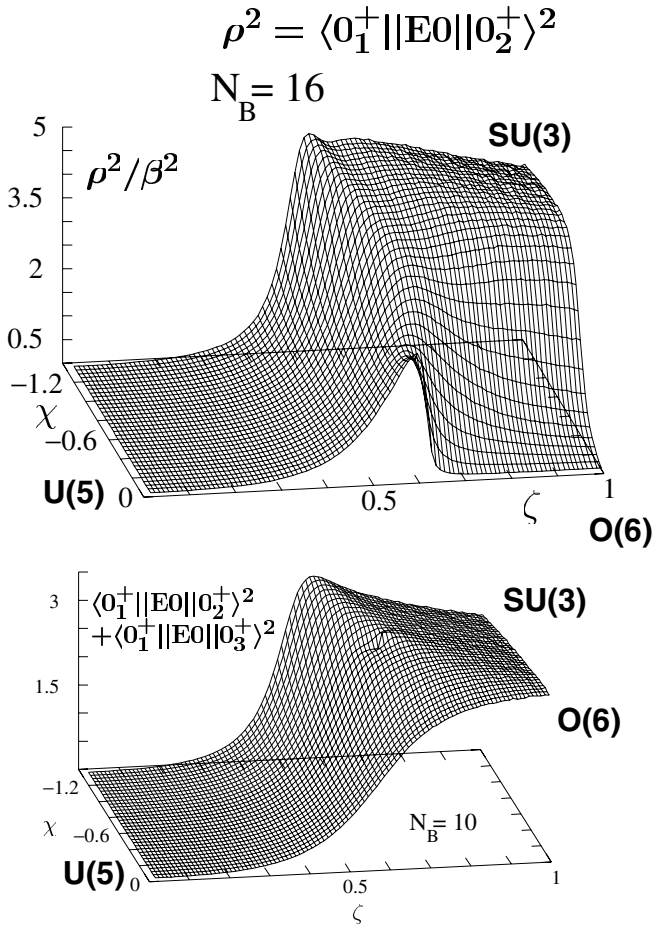


Fig. 1. Top: $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$ calculated for $N = 16$ bosons throughout the IBM parameter space. Bottom: the sum $\rho^2(E0; 0_2^+ \rightarrow 0_1^+) + \rho^2(E0; 0_3^+ \rightarrow 0_1^+)$ for $N = 10$ bosons.

energy ratio. The predicted rise in $E0$ strength between vibrator and rotor matches well the data. Nuclei for which $R_{4/2} < 2$ have not been considered as they are outside the model space.

4 Discussion

Earlier calculations [5] modeled large $E0$ strengths by using the Duval-Barrett formalism [6], mixing two model spaces. This seems to be conflicting with our approach using one model space only. However, ref. [5] used a very small mixing for the two model spaces, *e.g.*, in $^{96,102,104}\text{Mo}$, that means in a spherical ($A = 96$) and the first two deformed Mo isotopes. Therefore, these calculations effectively go over into the single space IBM results before and after the transition region. Only for $^{98,100}\text{Mo}$, for which the experimental values of the ratios $B(E2; 0_2^+ \rightarrow 2_1^+)/B(E2; 2_1^+ \rightarrow 0_1^+)$ and $B(E2; 2_2^+ \rightarrow 2_1^+)/B(E2; 2_1^+ \rightarrow 0_1^+)$ exceed any predictions of standard models, there is substantial mixing, and the Duval-Barrett formalism is required.

This shows that large $\rho^2(E0; 0_2^+ \rightarrow 0_1^+)$ values in transitional nuclei can arise either from mixing of coexisting

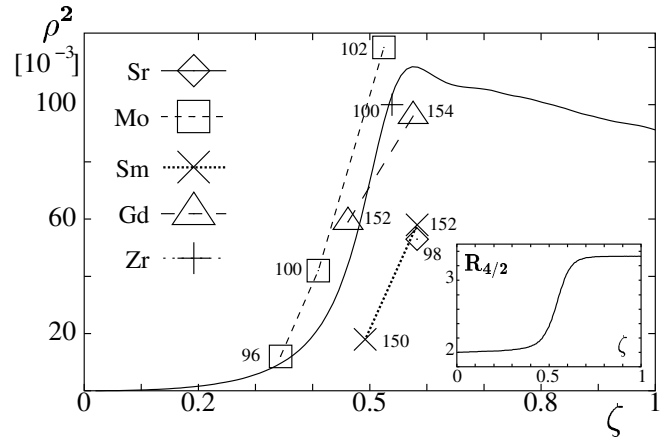


Fig. 2. Data compared to a schematic IBM-1 calculation. The insert gives the $R_{4/2}$ ratio to which ζ was fitted.

spherical and intruder configurations, or from the simpler IBM-1 itself. The key point is that large $E0$ values do not require a two-space mixing, but, in the first deformed nuclei in the mass 100 (^{98}Sr , ^{100}Zr , ^{102}Mo) and 150 (^{152}Sm , ^{154}Gd) regions, such large values are accounted for within the simple IBM-1 itself.

Microscopically, $E0$ transitions are forbidden in a single harmonic oscillator shell. However, realistic shell model descriptions effectively entail mixing of several oscillator shells, which should effectively be incorporated in the IBM, *e.g.*, by the use of effective charges. Thus, the $E0$ strengths in the IBM may reflect the fact that realistic major shells in the independent particle model include an intruder orbit from the next higher shell, and that additional intruder orbits appear in the Nilsson scheme with increasing deformation, that is, as the shape/phase transition proceeds. A detailed microscopic analysis would be needed to relate the IBM to such a picture. However, the appearance of intruder orbits may be reflected in the effective parameter β' in the $E0$ operator given in eq. (2), a simple one-body operator with constant parameters which, remarkably, is sufficient for reproducing the data in transition regions. The prediction that $E0$ strengths remain large in well-deformed rotors needs experimental confirmation.

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